A review on empirical mode decomposition in forecasting time series

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Abstract. This study aims to survey and summarize the studies that introduced forecasting time series method based on EMD, providing references for researchers relating to this topic. We highlight results that have published during $1998 - 2017$ (since presented the EMD technique). In this survey, we also present some studies that improved EMD methodology to overcome its limitations. In this survey, we present some studies that improved EMD methodology to overcome its limitations, as well studies that have introduced an expansion of EMD methodology. There has been tremendous progress in many areas, but we find that there are a large number of topics that need to be further developed. Finally, we summary some remarks may it will help the researcher in this area.

Keywords: empirical mode decomposition, intrinsic mode function, hybrid approach, forecasting time series, review.

1. Introduction

Forecasting time series is one of the core activities in scientific research, but limited by the availability of prediction methods in the past. To accommodate the variety of data generated by nonlinear and nonstationary processes in nature,

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the forecasting method would have to be adaptive. Hilbert-Huang transform (HHT), consisting of Empirical Mode Decomposition (EMD) and Hilbert spectral analysis [1], is a newly developed adaptive data analysis method, which has been used extensively in prediction research.

A number of studies in literature review that have introduced to review of EMD, such as [2], [3], [4], [5] and [6]. While, [2] presented a short review of EMD method as data analyzing method with example of its application on Earthquake data, [3] presented the recent developments until 2008 with summarize some applications in various geophysical research areas, [4] presented the principle of EMD and some of its extensions method with their application in image fusion, [5] presented review on EMD in fault diagnosis of rotating machinery and [6] presented review on EMD in fault diagnosis of rotating machinery diagnosis.

In this review, we will display the EMD methodology steps, a highlight on some of the studies that tried to develop on EMD method and introduced an expand of EMD method will be presented, with some studies that presented a comparison of EMD method with another decomposition methods. In addition, the aim of this study, the studies introduced forecasting time series method based on EMD. Moreover, for each forecasting technique, we present its methodology, the data used and the methods that have used in a comparison. We hope this study will be an assistant reference for interested and concerned researchers in this area in the future.

1.1 Hilbert-Huang transform

The Hilbert-Huang transform (HHT) has presented by [1] as a integrate of empirical mode decomposition (EMD) and Hilbert transform analysis (HT). The strength of HHT is the ability to process non-stationary and non-linear data. Moreover, HHT does not move from the time domain into the frequency domain - Information is maintained in the time domain [7]. While the HT is applies on intrinsic mode functions (IMF) along with a residual, and obtain instantaneous frequency data. Its mean, the component decomposition of signals must be performed beforehand before applied HT. Mathematically, the HT of a time series $x(t)$ is given by formula 1 in [1].

$$\mathcal{H}[x(t)] = \frac{1}{\pi} \text{ PV} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}\, d\tau \tag{1}$$

in which the PV indicates the principal value of the singular integral. In the next section, we will present a different analysis method from the previous methods that have been dealt with, while this method have a different components in its results (seasonal, trend and remainder) and also does not use the transformation technique in its algorithm. In this study the HT is not discussed, only the EMD procedure with applications in time series forecasting are presented.

2. Empirical Mode Decomposition (EMD)

The Empirical mode decomposition (EMD) is a new decomposition method has been described by [1]. The main idea for EMD is to decompose a non-stationary and nonlinear time series into a nearly orthogonal combination of simple time series [8]. These components known as intrinsic mode functions (IMFs) and residual (r). The EMD methodology analyze the time series with keeping this time series in time domain. This decomposition method (EMD) is adaptive, intuitive, direct and highly efficient. After this definition and some property of EMD, in the subsection of this section, the EMD algorithm process, several of application of EMD, theoretical developments on EMD, and forecasting methods based on EMD will be presented.

2.1 Sifting Process

The algorithm process of EMD will be presented as 6 steps. This process is named the Sifting process of EMD. While the main idea of EMD is decompose of time series into IMFs and $r(t)$. So, the time series -let $x(t)$- can be constructed back as the equation (2). The sifting decomposition process is based on the local characteristic time scale of the data as presented by [1].

$$x(t) = \sum_{i=1}^{n} IMF_i(t) + r(t), \tag{2}$$

where $x(t)$ represents the original time series, $r(t)$ represents the residue of the original time series data decomposition, and IMF_i represents the i^{th} intrinsic mode function (IMF) series. In order to estimate the IMFs should initiate the steps of sifting process of time series $x(t)$ as presented by [1]. Thus, this is summarized as described below:-

Step 1. The first step begins by taking the original time series as a $x(t)$ for sifting process. Also, we assume that the value of the two repetition indicators are $i = 1$ and $j = 1$.

Step 2. Then evaluating all the local extrima values (local upper and local lower) of the time series $x(t)$. Figure 1 shows an example of step 2. Here, the black line is the original time series $x(t)$, the red circle represents the local upper, and the green circle represents the local lower.

Step 3. After that, form the local upper (local maximum) envelope function $e_u(t)$ by connecting all local maxima values using the cubic spline line. In a similar way, form the local lower (local minimum) envelope function $e_l(t)$. Should all observations in $x(t)$ cover between $e_u(t)$ and $e_l(t)$. After that, form the mean envelop function denoted by $m_j(t)$ from $e_u(t)$ and $e_l(t)$ by using formula (3). Figure 2 shows an example of step 3. Here the black line represents the original time series $x(t)$, the red line represents the upper envelope line

$e_u(t)$, the green line represents the lower envelope line $e_l(t)$, and the blue line represents the mean envelop $m_j(t)$.

$$m_j(t) = \frac{e_u(t) + e_l(t)}{2} \tag{3}$$

Step 4. Next, define a new function $h_j(t)$ using the mean envelope $m_j(t)$ and the signal $x(t)$ on formula (4).

$$h_j(t) = x(t) - m_j(t) \tag{4}$$

Check if the function $h_j(t)$ which is an IMF or not, according to IMF conditions (will be presented in section 2.2). If the function $h_j(t)$ has satisfied IMF conditions, then go to step 5. If not, renew the value of $x(t)$ such that it become $h_j(t)$. Also, the iteration index value j is renewed. Such that it becomes $j = j + 1$, and repeat the steps again from step 2 until step 4.

Step 5. This step has three processes. Firstly, save $h_j(t)$ which obtained from the last step as a IMF_i, where $\mathrm{IMF}_i(t) = h_j(t)$. Secondly, obtain the residue function $r_i(t)$ using the $\mathrm{IMF}_i(t)$ and the signal $x(t)$ by the formula (5). Thirdly, renew the iterations index values of i and j. Such that it become $i = i + 1$ and $j = 1$.

$$r_{i+1}(t) = x(t) - IMF_i(t) \tag{5}$$

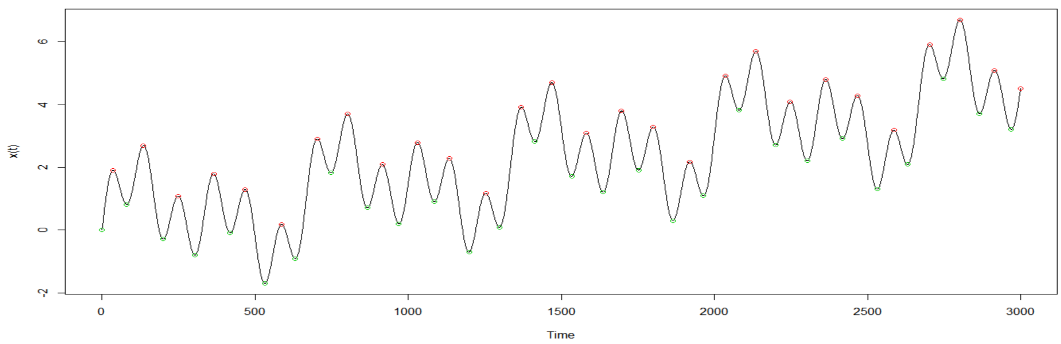

Figure 1: The extraction of local extremum values of time series

Step 6. In this step, according to the residual function's characteristics $r_i(t)$, which obtained from step 5, it will be decided whether the sifting process is over or not.

If $r_i(t)$ is monotonic or constant function from which cannot extract more IMF or the value of SD_h (standard deviation) between 0.2 and 0.3 by [1], where SD_h defined as formula (6), then the residue and all the IMF's will be saved,

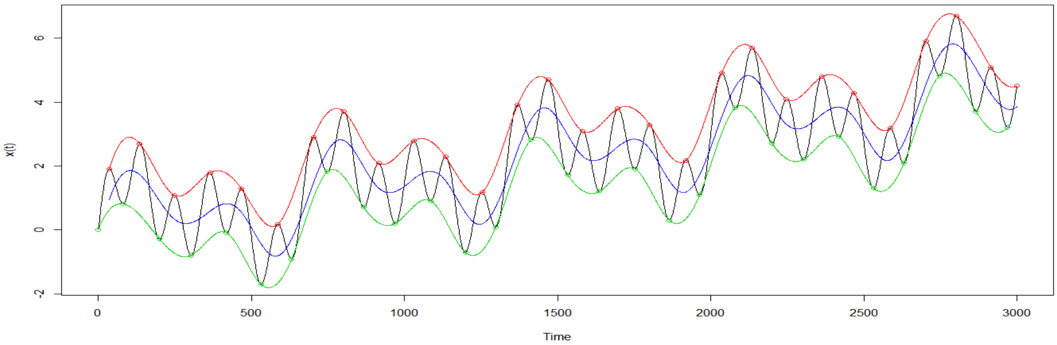

Figure 2: The evaluation of upper and lower envelopes of the time series.

and stop the sifting process. If the residue $r_i(t)$ is not, the value of $x(t)$ will be renewed, such that it becomes $r_i(t)$, and go back to step 2. This step is named "Stoppage criteria of the sifting process".

$$SD_h = \sum_{t=0}^{T} \frac{|h_{k-1}(t) - h_k(t)|^2}{h_{k-1}^2(t)}. \tag{6}$$

The steps 1 through 6 which were discussed above allow the sifting process (EMD algorithm) to separate the time-altering signal properties. Figure 3 is a flowchart summarizes all the sifting process steps.

2.2 Intrinsic Mode Function (IMF)

Based on the EMD algorithm which presented in the previous section, the IMFs' produced by the sifting process need to satisfy two conditions these are

1. $$|Num[extrima] - Num[cross - zero]| \leq 1.$$

   Where Num.extreme represents the number of local extreme points (all local maxima and all local minima), also Num[cross-zero] represent the number of cross-zero points.

2. $$|m(t)| = |\frac{e_u(t) + e_l(t)}{2}| < \varepsilon.$$

   Where $u(t)$ represents the envelope function generated by using cubic spline line on all local maxima, $l(t)$ represents the envelope function generated by using a cube spline line on all local minima, $m(t)$ represents the mean function that it was obtained by evaluating the mean of $u(t)$ and $l(t)$, and ε is a very small positive number which is close to zero. Sometimes, it is equal to zero.

The UK stock market data are taken as an example to show the original time series with its IMFs and residue. The results are displayed in Figure 4.

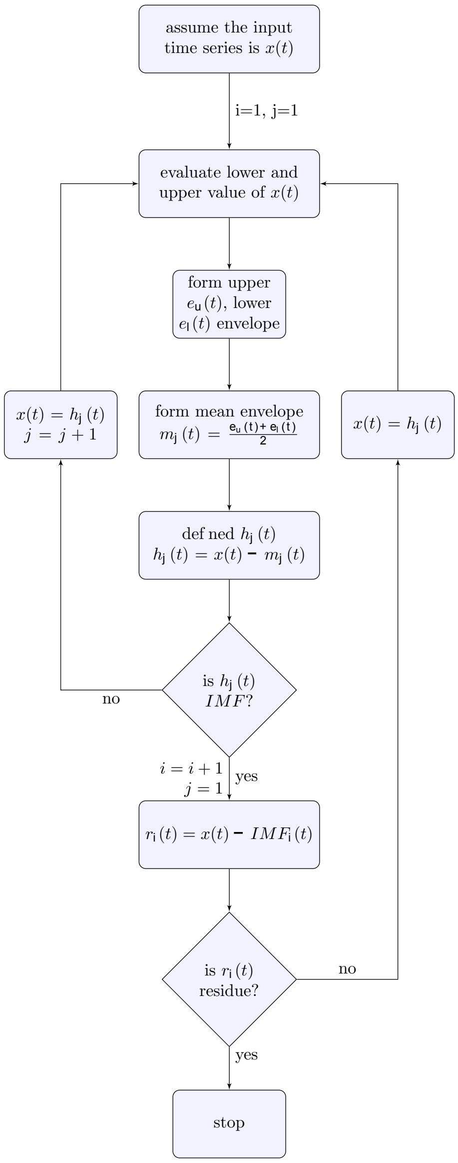

Figure 3: Flowchart of empirical mode decomposition estimation process.

2.3 Limitation, extension, comparison, and applications for EMD

After the introducing the EMD in the research field, it has been widely used in **application** for many research area. Such as, in financial time series by [**?**], [10]. In Medicine by [11]. In Mechanical Engineering by [12], the EMD method was employed in EMD-Golay de-noising algorithm to reduce the noise effectively on Lidar Signal. In electronics engineering by [13]. In sciences such as biological by [14], climate by [15], and dynamic by [16]. In civil & construction Engineering by [17]. Also, it has used in traffic by [**?**] and [19]. It is worth mentioning that

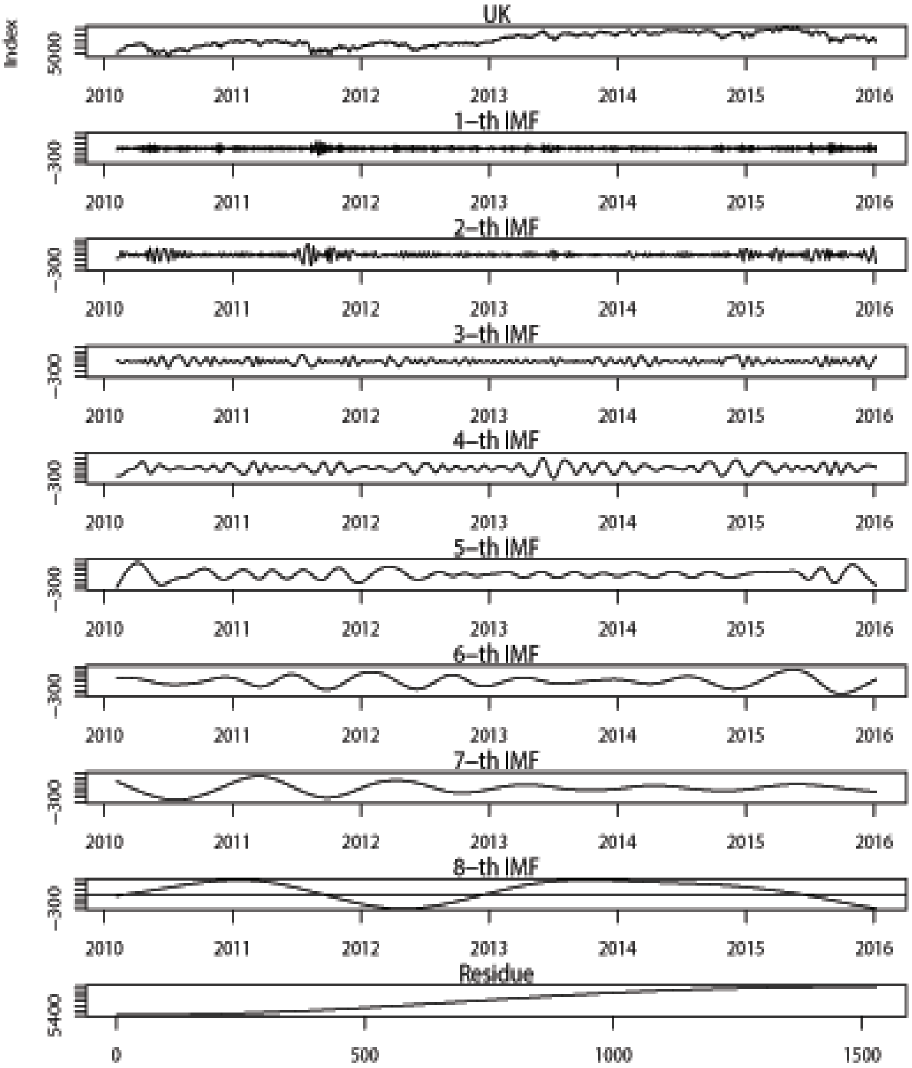

Figure 4: The UK stock market data with its IMFs and residue

the EMD technology has a number of **limitations** in its algorithm. The first is that the theoretical base has not fully established. While that the most of the EMD methodology steps without mathematical expressions. Moreover, [20] declared that there is no theory for EMD. Therefore, many studies have put some theoretical assumption for EMD, despite it still defined largely as steps of an algorithm. Such as, for a time series of size N, it usually only needs $\log_2 N$ IMFs [21]. Also, the average period of each IMF can be calculated by $\frac{2\times N}{(\# \, of \, zero \, crossings)}$ [22].

In [23], assumes that the IMF components are all normally distributed and the Fourier spectra of the IMF components are all identical and cover the same area on a semi-logarithmic period scale. [24] tried to develop the theoretical fundamentals of EMD algorithm by introducing three hypotheses on EMD sifting process. However, the theoretical part of the EMD is still poor [25].

The second limitation is the sensitivity to endpoint treatments (the boundary effect) when using the EMD algorithm [15]. To overcome this limitation, many studies have been developed on the EMD methodology. Such as [26] has extended both the beginning and end of the time series by the addition of typical waves. While, [27] companied local polynomial quantile regression (LLQ) with sifting process for automatic boundary correction.

The third limitation is mode mixing [28]. [29] is one of the studies that tried to solve this limitation, this by increasing amount of EMD iteration with additional mathematical operators based on differentiation and integration. Also, [30] by introducing an approach based on Partial Differential Equation as an alternative implementation to the algorithmic of the sifting process, they applied this technique on image analysis in [31]. Plus, [32] introduced a novel method based on the revised blind source separation, [33] applied the differential operation into the separation of the IMFs, and [8] introduced wavelet-bounded EMD to overcome the third limitation.

A number of studies have provided an **extension** of the EMD technology. Such as [34] generalize the EMD technique for two-dimensional. After that, [35] present multivariate extensions of EMD. Also, [36] present a fast three-dimensional EMD (TEMD) to decompose a volume into three-dimensional IMFs. On the other hand, [37] present Variational mode decomposition (VMD) as an alternative decomposition method to the EMD. In [38], the VMD has been applied to noise reduction of the diesel engine. In [39], the Complex VMD (CVMD) has been presented as a development of VMD to applied in Complex-valued signals.

In [40], the Ensemble EMD (EEMD) has been presented as an extension of EMD. The EEMD method has been applied in [41] and [42]. In [43], the complete EEMD (CEEMD) has been presented as a development of EEMD method. Moreover, the EMD algorithm has been modified by [44], [45], and [46].

Recently, a number of studies have presented a **comparison** between the EMD technique with another decomposition method. Such as, [47] present the comparison between EMD and Wavelet Decomposition (WD) in the nonlinear time series analysis. The authors deduce that the accuracy of the decomposition of the EMD is better than that Wavelet decomposition. Moreover, there is a difficult problem in the WD is the selection of the wavelet basis function and decomposition levels; while in the EMD there is none. Also, the authors have inferred that the EMD-based HT was decent for decomposing the linear and nonlinear regime. While the method of the WD-based WT was accurate for decompose the linear regime.

In [48], the EMD method was applied on Ultrasonic Signals for comparison with the Chirplet Signal Decomposition (CSD) method.Both these methods were applied to ultrasonic signal feature extraction with the aim of gaining more detailed feature information. The EMD method has accurate parameter estimation of time series. It is dynamically method to track the changing in time series.

2.4 Forecasting methods based on EMD

The hybrid models combine the strengths of some traditional models to get a better forecasting accuracy. Recently, several studies addressed hybrid models have applied EMD in the literature for time series forecasting. That by using EMD to decompose the non-stationary and non-linear time series data into Intrinsic Mode Functions (IMFs) and residual components. Then use the forecasting model to forecast each component. Later, the forecasting results were aggregated to get the final forecasted value of the original time series.

A hybrid EMD-ARIMA model has been used in [49, 50, 51, **?**] to forecast the short-term wind speed data, the monthly prices of rice data, the exchange rates data, and the traffic speed data, respectively. This by applying the ARIMA model on EMD components. Then aggregate all results of forecasting. In per of these studies, the forecasting results of the EMD-ARIMA model are superior to the forecasting results of the selected techniques.

A hybrid EMD-LSSVR (least squares support vector regression) forecasting model has been presented and applied on foreign exchange rate in [52]. The results show that the EMD-LSSVR model outperforms EMD-ARIMA, LSSVR, and ARIMA models. The methodology of EMD-LSSVR was by applying the LSSVR to forecast each component of EMD. Then all the forecasted values were aggregated to produce the final forecasted value.

In [53], the EMD-BPN (back-propagation neural network) was presented. All EMD components were modeled and forecasted by BPN. The final forecasting value can be obtained by the sum of these forecasting results. The EMD-NPN model was applied to wind power for short-term forecasting. The results show that the EMD-NPN model outperforms the BPN and ARIMA models.

In [54], the EMD-RBFNN (Radial basis function neural networks) was presented and applied on wind farm power. The results show that the EMD-RBFNN model has better forecasting accuracy than RBFNN model. The RBFNN forecasting model was built for each EMD component according to its feature. After that, all forecasting values were aggregated to obtain the final forecasting value.

In [55], the hybrid MFES model was presented and applied to forecast a half-hour electricity demand data. The MFES combines a multi-output FFNN (feedforward neural network) with EMD-based signal filtering and seasonal adjustment. The results demonstrated that the MFES model forecasting outcome was more accurate than MFE, MFS and MFES models.

In [56], the EMD-ANFIS (Adaptive Neuro-Fuzzy Inference System) model was presented. The ANFIS models were developed for EMD components. Then, these models were applied on IMFs to estimate the forecasting value. All the results were combined together to get the final forecasting. The EMD-ANFIS was applied to forecast Electric Peak Load data. The forecasting results of EMD-ANFIS model were more accurate than traditional Artificial Neural Network (ANN) models and the EMD-ANN.

[57] used a hybrid of EMD, Least Squares Support Vector Machine (LSSVM), and autoregressive (AR) to forecast wind speed data. The results show that the EMD-LSSVM-AR provides better forecasting compared with two hybrid model; the first model is the EMD-AR by [49] and the second model is EMD-LSSVM by [58]. The EMD components were classified into two sets (low and high correlated) according to the obtained partial autocorrelation function (PACF) factor and frequency. LS-SVM was applied on low correlated and AR model with Kalman filter model was applied on the highly correlated. The forecasting results were aggregated.

In [59], the EMD-ANN was proposed. This proposed method apply ANN on each EMD component before forecasting. The results were added together. The EMD-ANN model was applied on the stream flow data of river to compare with ANN model. The results show that the EMD-ANN model provided a superior alternative to the ANN model.

In [60], a new hybrid has been recommended and applied on uterine electromyography. The entropy ratios values of both instantaneous amplitude and instantaneous frequency of the first ten EMD components were computed. Six different classifiers were implemented in order to evaluate the forecasting performance. The results show an improvement in forecasting accuracy of compared with the existent techniques.

In [61], the integration of EMD and LSSVM model was used to forecast the water demand series data. LSSVM was built to forecast all EMD components individually, PACF was used as an input data. All of these forecasting values were then aggregated. The results show that the EMD-LSSVM model were better than the single LSSVM and ANN model without EMD and EMD-ANN model.

In [62], each EMD component represents the high and low frequencies as well as the patterns. Then meaningful signals were identified using Pearson product moment correlation coefficient. After the identification process was done, the new data set was obtained where the less meaningful signal was omitted from the signals sets. Then, a LSSVM was applied to forecast each IMF. This model was applied for river flow forecasting. The results was proven that EMD-LSSVM model outperforms a single LSSVM based on several performance criteria.

In [63], the integration of Phase space reconstruction (PSR), EMD, and ANN techniques (PSR-EMD-NNPSO) optimized by particle swarm optimization was applied to forecast stock index data.

In [64], the AR-EMD-SVR was presented. This model was employed to short-term forecast of ship motion time series. The time series was decomposed into IMFs and a residual by AR- EMD. The components was forecasted individually using SVR model. The forecasting results were aggregated. The AR-

EMD-SVR model results were better than forecasting results of AR, EMD-AR, and SVR models.

In [65], the EMD-SVR model was suggested. The attribute selection module was used to determine the parameter and compose the input vector for the SVR model of each EMD component. Then, the SVR module was trained and forecasted. Using wind speed data, the EMD-SVR model was compared with five forecasting models namely SVR, EMD-SVR by [66], EMD-SVR by [67], EMD-RBFNN by [68], and EMD-BP by [69]. The results show that the EMD-SVR model has significantly better performance than the selected method.

In [?], the EMD and Dynamic Regression (EMD-DR) was presented. Each EMD component was fit and forecasted with suitable DR model. Then the forecasting results were aggregated. The EMD-DR was employed for short-term load (electricity demand and reactive power) forecasting. The forecasting results from EMD-DR model was better than the forecasting results from a single Dynamic Regression model.

In [71], the hybrid EMD-AR model is presented. The AR model was applied on each EMD component to find the forecasting value. The results were aggregated to attain the final forecasting. The EMD-AR model was applied on ocean waves data. Forecasting results show that the hybrid EMD-AR model was superior to the AR model.

In [72], the DEMD-QPSO-SVR-AR model was presented and applied on two real electric load data to compare with ARIMA, BPNN, GA-ANN, PSO-BP, SVR, PSO-SVR, and, AFCM models. The differential EMD (DEMD) was applied on time series to decompose into a number of IMFs and residual. Quantum-Behaved Particle Swarm Optimization with support vector regression model (QPSO-SVR) was applied on each EMD component to find the forecasting value. Auto regression model (AR) was applied on residual to find the forecasting value. Then the final forecasting value were obtained from the IMFs and residual forecasting value.

In [73], an integrated forecasting model of EMD, ARIMA with SVR was presented. The original time series was decomposed into two part, linear and nonlinear. ARIMA was used to analyze and forecast the linear part. EMD was applied on the non-linear part, each component was forecasted by SVR model. Then, the forecasting values for all component were added to the forecasting value from the first part to get the final forecasting. This technique was applied to the stock index of four countries. The forecasting results of this model were better accuracy than ARIMA, SVR, EMD+SVR, ARIMA+SVR.

In [74], the DSF-ANN (Decomposition Selection Forecasting -ANN) and DSF-SVM forecasting models were proposed. For each EMD component, the initial features and targets were constructed. Then, a feature selection process was introduced to constitute the relevant and informative features. Then, the ANN or SVM model was built using the selected feature to evaluate the final forecasting. Three wind speed time series were used to compare these models with SVM, ANN, Decomposition Forecasting Aggregation-ANN (DFA-ANN) and DFA-SVM. The results show that these two models have satisfactory performance for the wind speed forecasting.

[75] presented EMD with radial basis function neural network (RBFNN) to forecast the monthly groundwater depth data. The RBFNN was used to forecast and stack each EMD separation sequence. The results showed that EMD-RBFNN model better than the conventional time series mode [76].

In [?], the modified EMD-LSSVM (MEMD-LSSVM) model was presented and used to forecast the exchange rate data. The EMD components were clustered into several groups based on Permutation distribution clustering (PDC). After that, LSSVM was used to train and forecast each group. All forecasting values added up together. The result shows that the MEMD-LSSVM outperforms single LSSVM and hybrid model of EMD-LSSVM. Moreover, the MED-LSSVM results were better than the MEMD-ARIMA result on the same data in [78]. In [79], a hybrid EMD with Holt-Winter method (HW) was applied to forecasting stock market data. The HW method was applied to forecast each EMD component, all forecasting results ware aggregated. Based on the three forecast accuracy measures, the results indicate that EMD-HW forecasting performance was superior to traditional HW forecasting method.

The combined EMD with exponential smoothing models (EMD-EXP) model was presented and applied to forecast stock market data in [80]. The EXP model was applied for forecast each EMD component. Then, all forecasting values were aggregated. The results show that EMD-EXP outperform four selected forecasting models based on five error forecasting measures.

In [81], a new combined TOPSIS-EMD-FNN model (Technique for order preference by similarity to an ideal solution, EMD, and ANN) was presented and applied on four time series. The TOPSIS-EMD reconstruction method was used to determine the weight for each component. The FNN was used to build a forecasting model for each component, all forecasting results were aggregated. Four forecasting methods were employed for comparison based on MSE, MAE, and MAPE. The results indicated that the TOPSIS-EMD-FNN method performs better than the other four models.

[82] applied the EMD with particle swarm optimization (PSO) and LSSVR model to forecast carbon price. The PSO-LSSVR was employed in forecasting the EMD components, the forecasting values of all the components were aggregated. The model's results were superior to four selected forecasting models.

In [83], a modified EMD-ANN model (MEMD-ANN) was applied to forecast tourism arrival data. The components produced via EMD by reconstructing some components through trial and error method (decomposition). This decomposition and the remaining these components were predicted using ANN model. The forecasted results were aggregated. The results show that, the MEMD-ANN outperformed the ANN and EMD-ANN models based on two measures. The same steps were applied in [84] with Group Method of Data Handling (GMDH) instead of ANN method. This model named MEMD-GMDH and was applied on tourism arrivals. The results showed that the EMD-GMDH model performed better than the traditional GMDH and EMD-GMDH [85] models based on RMSE and MAPE.

In [86], the integrated EMD with moving average (EMD-MA) model was presented and applied on stock market data. The MA model was applied for forecast each EMD component. Then, all forecasting values were aggregated. The results showed that EMD-MA outperform four selected forecasting models based on five error forecasting measures.

In [87], the hybrid model of EMD, phase space reconstruction, and extreme learning machine (EMD-PSR-ELM) was presented and applied to forecast exchange rates. The EMD components phase space was reconstructed to reveal its unseen dynamics according to the optimum time delay and embedding dimension. A regression forecast model was set up for each components by using ELM. All forecasting values were added up. The results show that the EMD-PSR-ELM superior than six existing method.

In [88], a combination EMD-SVM model was applied for forecast river flow data. The meaningful signals were identified for each EMD component using a statistical measure and the new dataset was obtained. After that, applied SVM to perform forecasting. The experiment results stated that the proposed EMD-SVM have outperformed selected models based on three measure.

In [89], the EMD and BPANN (back-propagation ANN) optimized by particle swarm optimization was presented. The three-layer BPANN was constructed to forecast each EMD component. Then, all forecasting results were aggregating. The outpatient visits data were used, the results showed that their method attains a better than the selected methods.

In [90], a hybrid of improved EEMD, ARIMA, extreme learning machine (ELM), and polynomial function (PF) were applied to forecast a hog price. Then, the EEMD components were composed into the high-frequency (HF), the low-frequency (LF), and trend terms. Then, the ELM, ARIMA, and PF were applied to forecast the HF, LF and trend terms, respectively. The forecasting results were aggregated. The results showed that the improved EEMD-ELM, ARIMA, PF approach outperforms the selected methods based on RMSE and SMAPE. In [92], a combination EEMD-ARIMA was applied to improving daily occupancy forecasting accuracy for hotels. The result showed that EEMD-ARIMA model improve accuracy compared to the ARIMA method.

In [**?**], a novel hybrid model of EMD with chaotic LSSVM (EMD-CLSSVM) was applied for annual runoff data. The LSSVM was applied to forecast the EMD components that possess chaotic characteristics, the rest were simulated by a polynomial method. The results were aggregated. The results reveal that the EMD-CLSSVM model better than the CLSSVM hybrid model based on RATED, RMSE, MARE, and MAE. In [93], EMD-HW bagging was used the EMD in bagging forecastingand applied on stock market data. The EMD with quantile regression (QR) were used to separate the data into regression line (RL), IMFs and residual. The IMFs were clustering into two clusters (HF and LF). Then the HF was resembling using a moving block bootstrap, then new HF series were add to LF, RL, and R. An ensemble of a hybrid EMD with HW model was applied to estimate the new series. The resulting point forecasts were combined by using the median. Based on three error measures, the results indicate that the EMD-HW bagging method outperforms seven forecasting models.

3. Conclusion

In this paper, we have attempted to provide a review of time series forecasting methods based on EMD. While, the EMD was effective to improve the forecast accuracy in all studies that have used it. Even after all the studies have done to improve the EMD, still there are open problems such as end effects, mode mixing, and Spline problem. Moreover, there is a lot of time series data need to improve its forecasting accuracy, because it is nonlinear and nonstationary, such as exchange rates time series.

Table 1 presents the summary of these literature review where contain the cite, Year , Methods are used and data are used. As a salvation to this part, there is no study was conducted using EMD with MA, EMD with HW, EMD with RW or EMD with EXP in the content of the stock market.

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Table 1: Related work used EMD in hybrid forecasting method.

	Cite	Year	Method	Data Category
1	[49]	2008	EMD-ARIMA	Wind speed
2	[52]	2012	EMD-LSSVR	Exchange rate
3	[53]	2012	EMD-BPN	Tourism demand
4	[54]	2012	EMD-RBFNN	Wind power
5	[55]	2013	EMD, FFNN	Electricity demand
6	[56]	2013	EMD, ANFIS	Peak Load
7	[57]	2013	EMD, LS-SVM, AR	Wind speed
8	[50]	2014	EMD-ARIMA	Prices of rice
9	[59]	2014	EMD-ANN	River stage
10	[51]	2015	EMD-ARIMA	Exchange rates
12	[61]	2015	EMD-LSSVM	Water demand
13	[64]	2015	AR-EMD-SVR	Ship motion
14	[?]	2015	EMD, ARIMA	Traffic speed
15	[60]	2015	EMD, novel nonlinear	Preterm Delivery
16	[62]	2015	EMD, LSSVM	River Flow
17	[63]	2015	EMD, PSR-EMD-NNPSO	Financial data
18	[65]	2016	EMD-SVR	Wind speed
19	[?]	2016	EMD, DR	Electricity demand
20	[71]	2016	EMD-AR	Ocean waves
21	[72]	2016	DEMD-QPSO-SVR-AR	Electric load
22	[73]	2016	EMD, SVR, ARIMA	Stock market
23	[74]	2016	EMD, SVM,ANN	Wind speed
24	[75]	2016	EMD, RBFNN	Groundwater depth
25	[?]	2016	MEMD, LS-SVM	Exchange Rate
26	[81]	2017	TOPSIS-EMD-FNN	Chaotic, temperature, sunspot, & flow
27	[82]	2017	EMD, LS-SVM	Carbon Price
28	[83]	2017	EMD, ANN	Tourism
29	[87]	2017	EMD, PSR, ELM	Exchange Rates
30	[88]	2017	EMD-SVM	River flow
31	[89]	2017	EMD and BPANN	outpatient visits
32	[90]	2017	iEEMD, ARIMA, ELM, PF	Hog price
33	[92]	2017	EEMD-ARIMA	Occupancy of hotels
34	[?]	2017	EMD,CLSSVM	Runoff

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